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ON THE OPTIMAL STOCK LEVELS IN A MULTI-ECHELON MAINTENANCE SYSTEM

US ARMY
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June 1978

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper develops bounds on the optimal stock levels in multi-echelon maintenance systems. These bounds can be used to find the optimal stock in multi-echelon systems such as METRIC and AMMIP. The use of the bounds in constrained multi-echelon problems is also demonstrated. An algorithm and computer code are presented.

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1. Introduction

In this report we present some useful bounds on the optimal stock levels in a multi-echelon maintenance system of the METRIC/AMMIP type [4,5]. A typical four echelon maintenance system is displayed in Figure 1.

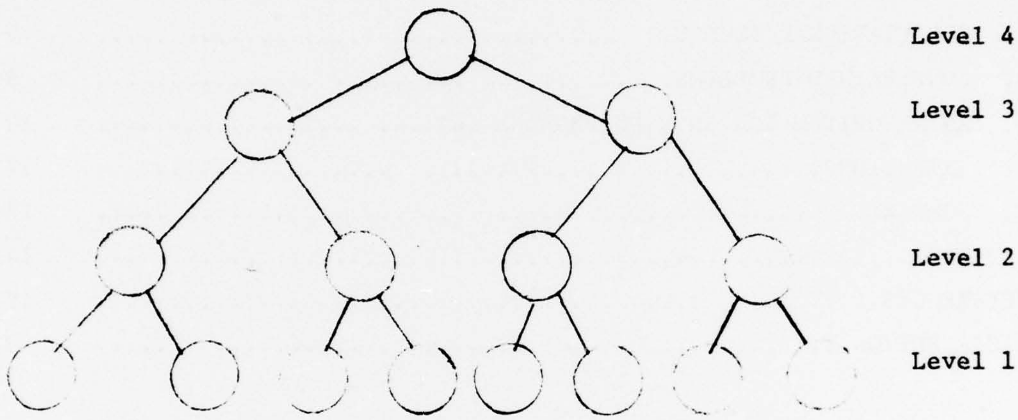

FIGURE 1

In a general n ($\geq$ 2) echelon maintenance system, the highest echelon (level n) is a lone installation referred to as the depot. Primary demands for reparable items occur only at the lowest level (level 1) installations. Depending on the nature of the repair required, the failed unit is either repaired at the level 1 installation to which it has been brought or is sent to a higher level location. At the higher level location, a decision is again made whether to repair the item there or to send the item to a higher echelon location for repair. All items received at the depot are repaired there. We assume that each time a decision is made on an item it is made independently of the decisions on other items. In many practical multi-echelon inventory systems, a level k location can send a failed item only to a level k+1 location if it does not repair the item at its own repair facility.

In either case we shall assume that all resupply is done on a one for one basis; that is, a continuous review (S-1,S) policy is followed. When a failed unit is sent to a higher level location for repair, the higher level location must resupply the lower level location with a serviceable unit when one is available for shipment.

The assumptions we make are described in detail in Sherbrooke [4] and Muckstadt [3]. Basically, we assume:

a. Demand at a level 1 location is a stationary (compound) Poisson process with rate λ_i at level 1 location i.

b. Repair times at an installation are independent and identically distributed.

c. A particular level k location always requests resupply from the same location on level k+j, j=1..., n-k when sending a failed item to that level.

d. There is no batching (or waiting) of items before repair is begun (infinite server assumption).

e. There is no lateral resupply among locations on the same level.

The problem we consider is to minimize the sum of the holding cost at each installation plus the time weighted backorder costs at all level 1 locations. Backorders at higher level installations are considered in the model only insofar as they affect performance at the level 1 locations.

A complete description of a very similar model can be found in Muckstadt [3]. We present here a brief description of the important concepts.

In order to compute the expected backorders outstanding at a point in time at a location, we need to know the expected resupply time for that installation. Let there be m_j installations on level j, j=1..., n-1, and let the installations be numbered consectively starting from 1 on the lowest level. Hence, the lowest level has installations $1,2,\ldots m_1$; level 2 has installations $m_1+1,\ldots, m_2+m_1$ etc. Let $N = \sum_{j=1}^{n-1} m_j$. Furthermore, let T_i = average resupply time for location i.
Say i is on level j. Then

$$(1) \qquad T_i = \gamma_i R_i + \sum_{k=j+1}^{n} r(i,k)\ [A_g^k + D_g^k]$$

where

R_i = repair time at installation i.

$r(i,k)$ = probability that a failed item at location i is sent to level k for repair

A_g^k = order and ship time between i and level k location g which is the unique level k location from which i seeks resupply.

D_g^k = delay at level k location g due to the unavailability of serviceable stock.

γ_i = probability that a failed item is repaired at i

$$= 1 - \sum_{k=j+1}^{n} r(i,k) = r(i,j)$$

By definition $T_{N+1} = T_o = R_o$ = depot repair cycle time.

The expected delay D_g^k is given by

$$D_g^k = \frac{\text{expected number of backorders outstanding at a point in time at level k location g}}{\text{average yearly demand at level k location g}}$$

Let N(g,k) = unique set of lower level installations that receive their resupply from level k location g when requesting resupply from a level k location, $k \geq 2$. Then λ_g = average yearly demand at this installation is given by

$$(2) \qquad \lambda_g = \sum_{j \in N(g,k)} r(j,k)\,\lambda_j \qquad g = m_1 + 1,\ldots,N+1$$

If S_g = stock level at installation g then the expected backorders at a point in time at this installation is given by

$$(3) \qquad \sum_{x>S_g} (x-S_g)\, P(x|\lambda_g T_g)$$

where

$P(x|\lambda_g T_g)$ = probability that x units are in resupply at location g given a lead time demand of $\lambda_g T_g$.

The overall problem we consider then is

$$\text{(4)} \qquad \text{Min } Z = C_H S_o + \sum_{j=1}^{N} C_H S_j + C_B \left(\sum_{k=1}^{m_1} \left(\sum_{x>S_k} (x-S_k) p(x|\lambda_k T_k) \right) \right)$$

where

C_H = annual holding cost per unit

C_B = backorder cost per unit year backordered

S_o = depot stock

Z is generally not a convex function so that optimization is not straightforward. Proposed solution schemes generally involve explicitly examining different possible stock levels in order to find the optimal solution. Hence, it is of paramount importance to be able to bound the value of the optimal stock levels at the various locations so that as few stock levels as possible need to be explicitly examined.

2. Computational Bounds

In this section we derive some useful bounds on the values of the optimal stock levels. Muckstadt [3] reported on some empirical bounds for a problem very similar to (4) for four echelons. His bounds were based on observations of optimal stock levels. Kruse [2] developed some theoretical results for (4) in a two echelon case. His results can be seen to be special cases of some of the results derived here.

Consider problem (4). If the depot stock, S_o, is fixed at some value y, solving (4) with $S_o = y$ is equivalent to solving the n-1 echelon problem.

$$\text{(5)} \qquad \text{Min } TCL(y) = \sum_{j=1}^{N} C_H S_j + C_B \left(\sum_{k=1}^{m_1} \left(\sum_{x>S_k} (x-s_k) p(x|\lambda_k T_k) \right) \right)$$

Note $Z = C_H y + TCL(y)$. Let $TCL^*(y)$ be the minimum total cost at the n-1 lower echelons when $S_o = y$. This is obtained by solving (5) with $S_o = y$. Then problem (4) can be restated as

$$\text{(6)} \qquad \text{Min } TC(S_o) = C_H S_o + TCL^*(S_o)$$

$$S_o = 0, 1, 2, \ldots$$

Let $TCL^*(\infty)$ = optimal cost at the lower echelons when there is never a delay at the depot due to unavailable serviceable stock when resupply is requested by a lower echelon location. The following Lemma is the crux of our major results.

Lemma: Let $j \leq k$. Then $TCL^*(j) \geq TCL^*(k)$

Proof: Let $B(S,T) = \sum_{k=1}^{m_1} \left(\sum_{x>S_k} (x-S_k)p(x|\lambda_k T_k)\right)$

This is an increasing function of T_k. Let $S^*(j)$ be the optimal lower echelon stock levels when $S_o = j$. By maintaining the same stock levels when $S_o = k$, the optimal lower echelon cost, $TCL^*(k)$ cannot increase since the holding cost at the lower echelons remains the same but backorder costs will not increase since T_k, $k = 1,\ldots,m_1$, cannot increase by adding more stock at the depot. Hence, by setting $S_o = k \geq j$ we can at least do as well at the lower echelons as when $S_o = j$. Hence, $TCL^*(j) \geq TCL^*(k)$.

Let S_o^* = value of the depot stock in an optimal solution to (4).

(7) Theorem 1: $S_o^* \leq \left[\dfrac{TCL^*(0) - TCL^*(\infty)}{C_H}\right]$

where $[x]$ denotes the largest integer $\leq x$.

Proof: Since S_o^* is the optimal depot stock, we have

$$TC(S_o^*) - TC(0) \leq 0$$

Thus, $C_H S_o^* + TCL^*(S_o^*) - TCL^*(0) \leq 0$

From the Lemma we have that $TCL^*(0) \geq TCL^*(S_o^*) \geq TCL^*(\infty)$ and hence

$$S_o^* \leq \frac{TCL^*(0) - TCL^*(\infty)}{C_H}$$

Since S_o^* must be an integer we have the desired result.

Corollary: Let $\bar{S}$ be such that $TC(\bar{S}) \leq TC(0) = TCL^*(0)$

Then

(8) $S_o^* \leq \left[\dfrac{TC(\bar{S}) - TCL^*(\infty)}{C_H}\right]$

provides a better bound than (7).

Proof: This follows as in the proof of Theorem 1 since

$$TC(S_o^*) - TC(\bar{S}) = C_H S_o^* + TCL^*(S_o^*) - TC(\bar{S}) \leq 0$$

Note that the bound (8) holds for any $\bar{S} = 0, 1, 2, \ldots$. However, (8) will be less than (7) if and only if $TC(\bar{S}) < TC(0)$.

We note as a consequence of the Lemma that $TCL^*(\infty)$ in (7) and (8) may be replaced with $TCL^*(k)$ as long as $k \geq S_o^*$. Computational experience has indicated that the delay experienced at the depot by an arriving unit decreases to zero rapidly as the depot stock level is increased beyond the value of the demand at the depot during a repair cycle. We replaced $TCL^*(\infty)$ by $TCL^*(k)$ for values of k that we felt were surely greater than S_o^*. For example, we used values for k equal to two times the depot lead time demand and another value of k equal to 5 + 1.1 times the depot lead time demand. However, it was noted that the $TCL^*(k)$ for these values of k were not significantly greater than $TCL^*(\infty)$ and the bounds (7) and (8) were not significantly improved. In fact, computational experience has indicated that $TCL^*(j)$decreases more rapidly for $j \leq S_o^*$ than for $j > S_o^*$. This result is not surprising. We would expect that adding more and more depot stock above S_o^* should have a decreasing affect on the optimal lower echelon stock levels and costs due to the rapid decrease in delay at the depot. Hence, in most computational procedures, we see no real advantage in replacing $TCL^*(\infty)$ in (7) or (8).

As mentioned in the previous section, the non-convexity of the minimization problem generally requires that various depot stock levels be explicitly examined. When solving problem (6) for a particular value of S_o it would be convenient to be able to use information obtained from previous calculations with other trial values of S_o. We now establish an upper bound on the amount of stock available for distribution to the lower echelons in an optimal solution. Let

$LEHC^*(k)$ = optimal lower echelon holding cost when $S_o = k$

$LEBC^*(k)$ = optimal lower echelon backorder cost when $S_o = k$

so that $TCL^*(k) = LEHC^*(k) + LEBC^*(k)$. Let $LES^*(k) = LEHC^*(k)/C_H$ = optimal total stock at all the lower echelon locations when $S_o = k$.

Theorem 2:

$$(9) \qquad LES^*(k) \leq [TCL^*(k-1)/C_H]$$

Proof: By the Lemma, $TCL^*(k) \leq TCL^*(k-1)$

$$LEHC^*(k) + LEBC^*(k) \leq TCL^*(k-1)$$

$$LES^*(k) \leq TCL^*(k-1)/C_H$$

Corollary: $LES^*(S_o^*) \leq [TCL(0)/C_H]$. In fact, if

$k \leq S_o^*$ then $LES(S_o^*) \leq [TCL(k)/C_H]$

The bound (9) is extremely useful when allocating stock among the lower echelons. Any allocation where the lower echelon stock allocated exceeds (9) can be ignored. Note that from (9) we have that

$$LES^*(k) \leq LES^*(k-1) + [\frac{LEBC^*(k-1)}{C_H}]$$

so that when $[\frac{LEBC^*(k-1)}{C_H}] = 0$, the total stock allocated to the lower echelons will not increase as depot stock goes from k-1 to k. However, this will not always be the case as one can construct examples where lower echelon stock will increase as S_o goes from k-1 to k.

Assume in (6) we are examining a particular value of S_o. Set the stock at all the level two through level n-1 locations at values higher than the values of the stock levels at these locations in an optimum solution. By doing this we have established resupply times to the level 1 locations $\underline{T}_i(S_o)$, i = 1,2,..., m_1 that are lower bounds on the actual optimal resupply times given depot stock is S_o. Given the $\underline{T}_i(S_o)$ as the resupply times we may calculate the stock levels for the level 1 locations $\underline{S}_i(S_o)$ that would be optimal if $\underline{T}_i(S_o)$ were the actual resupply times. Clearly, the $\underline{S}_i(S_o)$ are lower bounds on the actual optimal stock at the level 1

locations when depot stock is S_o. If $\sum_{i=1}^{m_1} \underline{S}_i(S_o) > [\frac{TCL(0)}{C_H}]$ then this particular value of depot stock cannot be optimal. One can employ this argument in a routine to determine a lower bound on S_o^*. If for $S_o = y$ we have $\sum_{i=1}^{m_1} \underline{S}_i(S_o) > [\frac{TCL(0)}{C_H}]$, then for any $j \leq y$ we must also have $\sum_{i=1}^{m_1} S_i(j) > [\frac{TCL(0)}{C_H}]$ since $\underline{T}_i(j) > \underline{T}_i(y)$. Hence, using a bisection type approach one can find a lower bound on S_o^*. There is a tradeoff between the computational cost of performing a search for a lower bound versus the cost of examining more depot stock levels explicitly. Generally, high demand situations where the demand at the depot during a repair cycle is large seems to be the only instances where establishing a lower bound on S_o^* may be advantageous.

3. Constrained Problems

The computational bounds developed above can be used in problems similar to (4). Consider (4) with a constraint on the total number of units that can be held (or equivalently a budget constraint). The new problem would be

$$(10) \qquad \text{Min } C_H S_o + \sum_{j=1}^{N} C_H S_j + C_B \left(\sum_{k=1}^{m_1} \left(\sum_{x>S_k} (x-S_k)p(x|\lambda_k T_k)\right)\right)$$

$$S_o + \sum_{j=1}^{N} S_j \leq \mu$$

By introducing a Lagrange multiplier, $\theta \geq 0$ (10) becomes

$$(11) \qquad \text{Min } C_H S_o + \sum_{j=1}^{N} C_H S_j + C_B\left(\sum_{k=1}^{m_1} \left(\sum_{x>S_k} (x-S_k)p(x|\lambda_k T_k)\right)\right)$$

$$+ \theta \left(S_o + \sum_{j=1}^{N} S_j\right)$$

Note for a fixed θ, (11) is equivalent to (4) where the holding cost for a unit has changed to $C_H + \theta$. Hence, in order to solve (10) one first solves the unconstrained problem. If the total optimal stock in the unconstrained

solution is $\leq \mu$ then we have an optimal solution to (10). If the total stock allocated exceeds μ, then we form the Lagrangian (11) and solve this problem. Solutions to (11) for particular values of θ yield undominated solutions and θ may be varied to obtain a satisfactorily close solution to (10).

Another constrained problem considered by authors [1] is

$$(12) \qquad \text{Min} \sum_{k=1}^{m_1} \sum_{x>S_k} (x-S_k)p(x|\lambda_k T_k)$$

$$S_o + \sum_{k=1}^{N} S_j \leq \mu$$

The constraint in (12) always holds at equality. By introducing a Lagrange multiplier $\theta \geq 0$, we obtain a problem similar to (4) but with the "holding cost" of the item now defined to be θ. Hence, the bounds obtained can be used in solving these types of problems as well.

4. An Algorithm for Solving Problem (4)

In this section we describe an algorithm for solving an n echelon problem. The algorithm takes full advantage of the computational bounds of Section 2.

We first fix the depot stock at some value, say $S_o = y$. When the depot stock is fixed, the resupply times for all the level n-1 locations have been determined. What remains is a number of n-1 echelon subproblems. The "depot" in each of these subproblems is a level n-1 location in the original problem. Hence, the results of the previous section apply to these new "depots". We solve these subproblems in the same manner as we solve the original problem. We fix the stock at the new "depots" and this stock, along with the fixed stock at the original depot, now determine the resupply times for all the level n-2 locations. We now solve a number of n-2 echelon subproblems. We continue in this manner until we are left with a number of two echelon subproblems which can be solved efficiently (see Kruse [2]).

Once we have obtained the solution to the two echelon subproblems, we work backwards and obtain a solution to the three echelon subproblems by

explicitly examining different stock levels on the third echelon. When the three echelon subproblems are solved we continue working backwards solving the four echelon subproblems, etc. until the n-1 echelon subproblems have been solved. We then have a solution to (6) for $S_o = y$.

The importance of the bounds derived in Section 2 can now be easily seen. In solving a typical n echelon problem, a number of n-1 echelon subproblems needs to be solved. Each of these involves the solution of a number of n-2 echelon subproblems, etc. Clearly, even for one value of depot stock a potentially large number of subproblems need to be solved. Hence, good bounds on optimal stock levels are useful tools in reducing the execution time of a computer program solving an n echelon problem.

To clarify the solution procedure we present a complete algorithm for a four echelon problem.

<u>Step 1</u>: Set $S_o = \infty$ and solve three echelon subproblems to get $TCL^*(\infty)$.

<u>Step 2</u>: Set $S_o = 0$ and find $TCL^*(0)$. Set $TCL^*(0)$ as incumbent minimum solution and save stock values.

<u>Step 3</u>: Calculate an upper bound on S_o^* using (7). Set $j = 1$.

<u>Step 4</u>: If $j >$ upper bound go to step 7. Otherwise, determine bound on total stock available for distribution to lower echelons ($= TCL^*(j-1)/C_H$) and find $TCL^*(j)$.

<u>Step 5</u>: If $TCL^*(j) + C_H\, j <$ incumbent set incumbent to $TCL^*(j) + C_H\, j$ and save stock levels. Update the upper bound on S_o^* using (8).

<u>Step 6</u>: Set $j = j + 1$ and go to step 4.

<u>Step 7</u>: Print out optimal solutions.

To solve the three echelon subproblems, we use the same algorithm as above with the word three replaced by two.

The logic used in the algorithm can be extended similarly to include any number of echelons.

5. Extensions

For ease of exposition we have assumed that primary demands occur only at level 1 locations. This restriction can easily be removed. Let W be the set of all locations at which primary demands occur. Then problem (4) becomes

$$\text{Min } Z = C_H S_o + \sum_{j=1}^{N} C_H S_j$$

$$+ C_B \left(\sum_{j\varepsilon W} \sum_{x>S_j} (x-S_j)p(x|\lambda_j T_j)\right)$$

The λ_g, $g\varepsilon W$ calculated in (2) need to be updated reflecting the fact that removals many occur at higher level locations.

Conceptually, one can view higher level locations at which removals occur as consisting of two parts - an operating activity and a repair activity. At the operating activity, the items are used and hence fail. The repair activity repairs failed items from the operating activity and failed items that are sent by lower echelon locations. It is not hard to see that it could never pay to designate stock at such a location just for use by the operating activity. Hence, all stock at such a location is assigned to the repair activity. Thus, every demand at the operating activity is backordered. However, if the repair activity has a serviceable unit on hand, resupply is instantaneous. Otherwise, the resupply time is just the expected delay until a serviceable unit becomes available at the repair activity since all resupply to the operating activity comes from the repair activity.

If we let

λ_j^1 = removals at level k ($\geq$ 2) location j operating activity

$$\lambda_j^2 = \sum_{y\varepsilon N(j,k)} r(y,k)\lambda_y$$

$\lambda_j^R = \lambda_j^1 + \lambda_j^2$ = average yearly demand at the repair activity

Then problem (4) can be written as

$$\text{Min } Z + C_H S_o = \sum_{j=1}^{N} C_H S_j$$

$$+ C_B \left(\sum_{j=1}^{m_1} \sum_{x>S_j} (x-S_j)p(x|\lambda_j T_j) \right)$$

$$+ C_B \sum_{j\epsilon W/\{1,2,\ldots m_1\}} (\lambda_J^1 T_J^A)$$

where T_J^A = resupply time for the operating activity at location j = expected delay at the repair activity of location j before a serviceable unit becomes unavailable

$$= \left(\sum_{x>S_j} (x-S_j)p(x|\lambda_j^R T_j) \right)$$

The bounds developed in section (3) can be applied to this problem with minor modification.

6. Summary

In this report we have presented some useful bounds on optimal stock levels in multi-echelon maintenance systems. The bounds were then incorporated into an algorithm for solving a specific n echelon problem. The true usefulness of the bounds depends heavily on the specific problems being investigated. Furthermore, in specific problems other factors may be present that provide bounds on the optimal stock levels. However, in most problems we feel the bounds presented here are extremely useful and they have already been successfully implemented on some multi-echelon maintenance problems. In the Appendix, we present a computing listing of the Multi Echelon Stockage Subroutines (MESS) developed at the US Army Inventory Research Office. The listing is a wording of the algorithm in Section 4 assuming that the mean demand at each level 1 location is the same and that a level k-1 location can send a failed item only to a level k location for repair if it does not repair the item at its own repair facility. These restrictions can easily be removed and the extensions of Section 5 easily incorporated to make the program more robust.

APPENDIX

```
      SUBROUTINE FOURECH(DLDEM,TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,OPSTOCK,MINCOST,AVAIL)
      DIMENSION STOCK(3),TAT(4),ORGSPER(4),DST(4),P(4),
     XOPSTOCK(4),PSUM(4),XLB(4)
      REAL LBTLEC,MINCOST
      RLTD(X)=BLTD+DEM3*BACKO(DLDEM,X)/DEM4
      TEMP=REMOVES*ORGSPER(1)*ORGSPER(2)
      DEM3=TEMP*(1.-PSUM(3))
      DEM4=DEM3*ORGSPER(3)
      BLTD=TEMP*(P(3)*TAT(3)+(1.-PSUM(3))*DST(3))
      CALL THREECH(BLTD,TAT,DST,P,PSUM,REMOVES,ORGSPER,CRATIO,
     XECHSTK,XLB,STOCK,CM,AVAIL)
      LBTLEC=CM*ORGSPER(3)
C     PRINT *,"LBTLEC=",LBTLEC
      CALL THREECH(RLTD(XLB(4)),TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,STOCK,CM,AVAIL)
      UBTC=XLB(4)*CRATIO+CM*ORGSPER(3)
C     PRINT *,"UBTC=",UBTC
      MINCOST=UBTC
      OPSTOCK(4)=XLB(4)
      OPSTOCK(3)=STOCK(3)
      OPSTOCK(1)=STOCK(1)
      OPSTOCK(2)=STOCK(2)
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      J=XLB(4)
 10   J=J+1
      IF(UPBOUND .LT. J)GO TO 100
      ECHSTK3=(AMIN1(ECHSTK-J,CM*ORGSPER(3)/CRATIO))/ORGSPER(3)
      CALL THREECH(RLTD(FLOAT(J)),TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK3,XLB,STOCK,CM,TAVAIL)
      COST=(CM*ORGSPER(3))+(J*CRATIO)
      IF(COST .GT. MINCOST)GO TO 10
C     PRINT*,"UBTC= ",UBTC
      UBTC=COST
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      MINCOST=COST
      OPSTOCK(4)=J
      OPSTOCK(3)=STOCK(3)
      OPSTOCK(2)=STOCK(2)
      OPSTOCK(1)=STOCK(1)
      AVAIL=TAVAIL
      GO TO 10
 100  RETURN
      END
```

```
      SUBROUTINE THREECH(DLDEM,TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,OPSTOCK,MINCOST,AVAIL)
      DIMENSION OPSTOCK(3),STOCK(2),DST(4),ORGSPER(4)
      DIMENSION TAT(4),P(4),PSUM(4),XLB(4)
      REAL LBTLEC,MINCOST
      RLTD(S)=BLTD+DEM2*BACKO(DLDEM,S)/DEM3
      DEM2=REMOVES*ORGSPER(1)*(1.-PSUM(2))
      DEM3=DEM2*ORGSPER(2)
      BLTD=REMOVES*ORGSPER(1)*(P(2)*TAT(2)+(1.-PSUM(2))*DST(2))
      CALL TWOECH(BLTD,TAT,DST,P,PSUM,REMOVES,ORGSPER,CRATIO,
     XECHSTK,XLB,STOCK,CM,AVAIL)
      LBTLEC=CM*ORGSPER(2)
      CALL TWOECH(RLTD(XLB(3)),TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,STOCK,CM,AVAIL)
      UBTC=XLB(3)*CRATIO+CM*ORGSPER(2)
C     PRINT *,"UBTC",UBTC
      MINCOST=UBTC
      OPSTOCK(1)=STOCK(1)
      OPSTOCK(2)=STOCK(2)
      OPSTOCK(3)=XLB(3)
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      J=XLB(3)
 10   J=J+1
      IF(UPBOUND .LT. J)GO TO 100
      ECHSTK2=AMIN1(ECHSTK-J,CM*ORGSPER(2)/CRATIO)/ORGSPER(2)
      CALL TWOECH(RLTD(FLOAT(J)),TAT,DST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK2,XLB,STOCK,CM,TAVAIL)
      COST=CM*ORGSPER(2)+(J*CRATIO)
      IF(COST .GT. MINCOST)GO TO 10
      UBTC=COST
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      MINCOST=COST
      OPSTOCK(3)=J
      OPSTOCK(2)=STOCK(2)
      OPSTOCK(1)=STOCK(1)
      AVAIL=TAVAIL
      GO TO 10
 100  RETURN
      END
      FUNCTION BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
      REAL LBTLEC
      TEMP=(UBTC-LBTLEC)/CRATIO
      BOUND=AMIN1(TEMP,ECHSTK)
      RETURN
      END
```

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```
      SUBROUTINE TWOECH(DLDEM,TAT,OST,P,PSUM,REMOVES,ORGSPER,
     XCRATIO,ECHSTK,XLB,STOCK,MINCOST,AVAIL)
      REAL NRAR,MOPT,LBTLEC,MINCOST
      DIMENSION STOCK(2),TAT(4),P(4),PSUM(4),OST(4),ORGSPER(4)
      DIMENSION XLB(4)
      RLTD(I)=BLTD+DEM1*BACKO(DLDEM,S)/DEM2
      DEM2=REMOVES*ORGSPER(1)*(1.-P(1))
      DEM1=DEM2/ORGSPER(1)
      BLTD=REMOVES*(P(1)*TAT(1)+(1.-P(1))*OST(1))
      NRAR=AINT(BLTD*2+5.)
      NRAR=ONEECH(CRATIO,RLTD(XLB(2)),NRAR,XLB(1))
      UBTC=ORGSPER(1)*(NRAR*CRATIO+BACKO(RLTD(XLB(2)),NRAR))+XLB(2)
     X*CRATIO
      STOCK(1)=NRAR
      MOPT=XLB(2)
      MINCOST=UBTC
      NRAR=ONEECH(CRATIO,BLTD,NRAR,XLB(1))
      LBTLEC=ORGSPER(1)*(NRAR*CRATIO+BACKO(BLTD,NRAR))
      NRAR=STOCK(1)
      J=XLB(2)
      UPBOUND=BOUND(LBTLEC,UBTC,ECHSTK,CRATIO)
200   J=J+1
      IF(UPBOUND .LT. J)GO TO 20
      STOCK(2) = J
      NRAR=ONEECH(CRATIO,RLTD(STOCK(2)),NRAR,XLB(1))
      COST=J*CRATIO+ORGSPER(1)*(NRAR*CRATIO+BACKO(RLTD(STOCK(2)),NRAR))
      IF (COST.GT.MINCOST) GO TO 200
      UBTC=COST
      MINCOST = COST
      STOCK(1) = NRAR
      MOPT = STOCK(2)
   15 GO TO 200
20    STOCK(2) = MOPT
      AVAIL=POISLT(RLTD(MOPT),STOCK(1))
      RETURN
      END
      FUNCTION ONEECH(CRATIO,PIPE,INITST,XLOBND)
      REAL NRAR, INITST
C
      NRAR = INITST
      STK = NRAR
      COSTAL = BACKO(PIPE,NRAR) + CRATIO*NRAR
      DO 10 I = 1, 100
      NRAR = NRAR - 1.
      IF (NRAR.LT.XLOBND) GO TO 20
      COSTAR = BACKO(PIPE,NRAR) + CRATIO*NRAR
C     ASSUME CONVEXITY
      IF (COSTAR.GE.COSTAL) GO TO 20
      STK = NRAR
      COSTAL = COSTAR
   10 CONTINUE
      PRINT *, "ONEECH INITST TOO BIG"
   20 ONEECH = STK
      RETURN
      END
```

```
      FUNCTION POISLT(P, S)
C***  POISLT - LEFT TAIL OF POISSON WITH MEAN P   (PROB<=S)
C
      LT = 1
      POISLT = 0.
      IF (S.LT.0.) RETURN
      GO TO 1
C
      ENTRY BACKO
C***  BACKO  - BACKORDERS (EXPECTED VALUE > S) HADLEY&WHITIN APP 3 #10
      LT = 2
      POISLT = P
      IF (S.LE.0.) RETURN
      GO TO 1
C
      ENTRY POISRT
C***  POISRT - RIGHT TAIL OF POISSON WITH MEAN  P   (PROB>S)
      LT = -1
      POISLT = 1.
      IF (S.LT.0.) RETURN
      GO TO 1
C
      ENTRY POISDN
C***  POISDN - POISSON DENSITY WITH MEAN P, STOCK S
      LT = 0
      POISLT = 0.
      IF (S.LT.0.) RETURN
    1 CONTINUE
      NS = S + .000001
      POIS = POISLT = EXP(-P)
      IF (NS.LT.1) GO TO 20
      DO 10 IS = 1, NS
      POIS = POIS*P/IS
   10 POISLT = POISLT + POIS
   20 IF ((S-NS).LT. .001) GO TO 30
      POISS2 = POIS*P/(NS+1)
      POISL2 = POISLT + POISS2
      POIS = (S-NS)*POISS2 + ((NS+1)-S)*POIS
      POISLT = (S-NS)*POISL2 + ((NS+1)-S)*POISLT
C     ABOVE STMTS FOR INTERPOLATION ON VALUE OF STOCK
   30 IF (LT.EQ.1) GO TO 40
C     BACKORDERS
      IF (LT.EQ.2) POISLT = P*POIS + (P-S)*(1.-POISLT)
C     RIGHT TAIL
      IF (LT.EQ.-1) POISLT = 1. - POISLT
C     DENSITY
      IF (LT.EQ.0) POISLT = POIS
   40 RETURN
      END
..
```

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